Physics 222 UCSD/225b UCSB

Lecture 4

• Weak Interactions (continued)
  • Neutrino scattering
  • Unitarity bound
  • GIM mechanism
Neutrino Electron Scattering

\[ \nu \equiv \bar{u}(k) \quad e^- \equiv u(p') \]
\[ \bar{\nu} \equiv \bar{v}(k) \quad \bar{v} \equiv v(k') \]
\[ e^- \equiv \bar{u}(p) \quad \nu \equiv u(k') \]
\[ e^- \equiv \bar{u}(p) \quad e^- \equiv u(p') \]

NC is different, e.g. it couples muon neutrinos to electrons, i.e. across flavor.
Historical aside

• If you want to look up the discovery of neutral currents, you might want to start here:
  
  http://cerncourier.com/cws/article/cern/29168

The basic challenge was to distinguish:

\[ \nu_\mu N \rightarrow \nu_\mu X \]

\[ \nu_\mu N \rightarrow \mu^- X \]

Where \( N = \text{nucleon}, \) and \( X = \text{hadron}. \)

For neutral current, there is no charged lepton in the final state.
For antineutrino scattering, the spin of the two incoming particles must couple via V-A because they attach at same vertex.

As a result, only one of the three possible spin combinations is allowed, and we get:

$$\sigma(\nu_e e^-) = 3\sigma(\bar{\nu}_e e^-)$$
Neutrino Electron Scattering

• Matrix Element for CC neutrino-electron:

\[ M = \frac{G}{\sqrt{2}} [\bar{u}(k')\gamma^\mu(1-\gamma^5)u(p)][\bar{u}(p')\gamma_\mu(1-\gamma^5)u(k)] \]

• Cross Section:

\[ \sigma(\nu_e e^{-}) = \frac{G^2 s}{\pi} \]

What does it mean for a cross section to increase with center of mass energy?

It’s a sign of a “low energy effective theory”!
Overview of “Unitarity bound” Discussion

• Use partial wave analysis to derive the largest possible cross section, $\sigma(s)$, that is compatible with probability conservation.
• Compare this with the calculated cross section.
• Calculate the center of mass energy scale, beyond which the 4-fermion interaction clearly makes no sense because it violates probability conservation.
• Show how the introduction of the W propagator helps to resolve this problem.
• Comment that even with W, issues remain that are related to the longitudinal W polarization.
• Hint that this is fixed only by requiring Gauge Symmetry.
Partial waves
(to review this, see e.g. Sakurai QM Chapter 7.6)

\[ \Psi_i = e^{ikz} = \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1)[(-1)^l e^{-ikr} - e^{ikr}] P_l(\cos \theta) \]

\[ \Psi_{total} = \Psi_{scattered} + \Psi_i = \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1)[(-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr}] P_l(\cos \theta) \]

Absorption coefficient = 1
If no energy is absorbed.
Allow for arbitrary phase shift.

Regroup to get scattered wave:

\[ \Psi_{scattered} = \Psi_{total} - \Psi_i = \frac{e^{ikr}}{kr} \sum_{l=0}^{\infty} (2l+1) \frac{\eta_l e^{2i\delta_l} - 1}{2i} P_l(\cos \theta) \]

\[ \Psi_{scattered} = \frac{e^{ikr}}{r} F(\theta) \]
Relate to Cross Section

- Scattered outgoing Flux $d\Omega = v_{out} \Psi_{scat} \Psi^*_{scat} r^2 d\Omega$

\[
F_{out} = v_{out} |F(\theta)|^2 d\Omega
\]

\[
F_{in} = \Psi_{in} \Psi^*_{in} v_{in} = v_{in}
\]

- Cross section:

\[
d\sigma = \frac{F_{out}}{F_{in}} = |F(\theta)|^2 d\Omega
\]

\[
\sigma = \int |F(\theta)|^2 d\Omega
\]

\[
\sigma = \frac{1}{k^2} \sum_{l,m} (2l+1) \left[ \frac{\eta_i e^{2i\delta_l}}{2i} - 1 \right](2m+1) \left[ \frac{\eta_m e^{2i\delta_m} - 1}{2i} \right]^* \times \int P_l(\cos \theta) P_m(\cos \theta) d\Omega
\]

\[
\sigma = \frac{4\pi}{k^2} \sum_l (2l+1) \left[ \frac{\eta_i e^{2i\delta_l}}{2i} - 1 \right]^2 = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l
\]
Conclusion from Partial wave excursion

• For our neutrino scattering we had:

\[
\frac{d\sigma(v_e e^-)}{d\Omega} = \frac{G^2 s}{4\pi^2}
\]

• The angle independence means that only s-wave (l=0) contributes, and we thus have the general bound:

\[
\sigma \leq \frac{4\pi}{k^2}
\]

=> Probability conservation is violated at:

\[
\sqrt{s} = k \approx 300 GeV
\]
Including the W propagator

• The 4-point Fermi theory thus violates s-matrix unitarity at $O(100\text{GeV})$ energy.
• If we include W propagator the point where s-matrix unitarity is violated is pushed out to $O(10^{11}) M_W$.

• However, a number of other problems remain!
Example WW production

• If you calculate WW production in neutrino scattering, you find:

\[ \sigma(\nu\nu \rightarrow W^+_L W^-_L) \xrightarrow{k^2 \rightarrow \infty} \left[ \frac{g}{M_w} \right]^4 \]

• While production of transversely polarized W’s remains constant.

• Clearly, here’s something problematic about longitudinally polarized massive vector bosons.
Resolving this in QED

• As an aside, the same problem does not arise for virtual photons in QED because of gauge invariance.
  – For more detailed discussion see Leader & Predazzi, Chapter 2.1
Aside on Gauge Invariance

• If we were to try and figure out a way to impose gauge invariance to weak interactions, we’d be tempted to postulate that $g \sim e$.

• It turns out that this works out quantitatively surprisingly well:

$$M_W = \left( \frac{\sqrt{2}g^2}{G} \right)^{\frac{1}{2}} \approx \left( \frac{\sqrt{2}e^2}{G} \right)^{\frac{1}{2}} \approx 106 GeV$$

• We’ll see later how to work this out correctly.
Conclusions

• Fermi Theory breaks down at high energies
  – This is a general feature of theories with dimensionful couplings.
• Including W by hand improves high energy behaviour, but does still leave problems, e.g. with vv -> WW for longitudinally polarized Ws.
• Problem with W seems to be related to longitudinal polarization.
• Might be fixed if we could construct a gauge theory of weak interactions.
• We find the EM & Weak almost unify in the most naïve way by setting e = g, and calculating the W mass correctly to within 10%.

Sounds like we are on to something!
GIM Mechanism

• Problem:

\[ \Gamma(K^+ \rightarrow \mu^+ \nu) = 5.1 \times 10^7 \text{ sec} \]
\[ \Gamma(K^0 \rightarrow \mu^+ \mu^-) = 1.4 \times 10^{-3} \text{ sec} \]

• First obvious conclusion:

This diagram doesn’t work -> Z couples to same flavor.
GIM Mechanism (2)

• 2nd order diagram is not sufficiently small

\[
\begin{array}{cc}
  \bar{s} & W \\
  d & u & W & \nu & e^+ \\
\end{array}
\]

• Glashow-İlliapolous-Maiani suggested that a c-quark exists, providing another 2nd order diagram to destructively interfere with first.

\[
\begin{array}{cc}
  \bar{s} & W \\
  d & c & W & \nu & e^+ \\
\end{array}
\]
GIM Mechanism (3)

• How do we arrange the destructive interference?

Old current:

\[ J^\mu = \cos \theta \ \bar{u} \gamma^\mu (1 - \gamma^5) d + \sin \theta \ \bar{u} \gamma^\mu (1 - \gamma^5) s \]

New current:

\[ J^\mu = \cos \theta \ \bar{u} \gamma^\mu (1 - \gamma^5) d + \sin \theta \ \bar{u} \gamma^\mu (1 - \gamma^5) s \]

\[ + \cos \theta \ \bar{c} \gamma^\mu (1 - \gamma^5) d - \sin \theta \ \bar{c} \gamma^\mu (1 - \gamma^5) s \]
GIM Mechanism (4)

To make this less ad-hoc, propose weak doublets, and a current that is diagonal, with a unitary mixing matrix between doublets to translate from weak to mass eigenstates:

\[
J_\mu^+ = (\bar{\nu}_e \nu_{\mu L} \bar{\nu}_{\tau L}) \gamma_\mu \begin{pmatrix} e_L^- \\ \mu_L^- \\ \tau_L^- \end{pmatrix} + (\bar{u}_L \bar{c}_L \bar{t}_L) \gamma_\mu V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}
\]
Concise statement of GIM

\[ \sum d'_i d'_i = \sum d_i U_{ij}^{T*} U_{jk} d_k = \sum d_i d_i \]

- The neutral current couples to the q’ (weak eigenstates) not the q (mass eigenstates).
- The matrix U is unitary because the weak coupling is universal, i.e. same for all weak eigenstates.
- As a result, only flavor conserving neutral currents are allowed.
Historic Aside

• From this we then find that the K0 to mu+mu- decay is absolutely forbidden if c and s masses are identical.

• In the literature, you sometimes find claims like: “From the observed rate, one could predict the charm quark mass to be 1-3GeV before its discovery.”

• Not sure this is true, and if true, then they were plain lucky because depending on $V_{cs} V_{cd}$ and $m_c$ versus $V_{ts} V_{td}$ and $m_{top}$, they could have been way off!
Outlook on next few lectures

• Next 3 lectures are on heavy flavor physics and CP violation.
  – Mixing phenomenology
    • 2-state formalism in its entirety, maybe as a homework.
  – CP violation in B-system
    • Categorize CP phenomenology
      – CP violation in decay, mixing, and interference between decay and mixing.
  – Experimental aspects of this subject
    • Measuring sin2beta @ Y4S
    • Measuring Bs mixing @ Tevatron
    • The future of this field: LHC-B and SuperBelle
After that we have choices:

- We can do the topics in two orders. Either way we will cover both:
  - Move on to SUSY for 2-3 lectures, and finish quarter off with EWK symmetry, higgs, et al.
  - Continue with EWK symmetry, higgs, et al., and do SUSY last.

- I don’t have much of a preference myself.
  - It makes more sense to do Standard Model first.
  - However, to start preparing for your seminar talk, it might be useful for me to talk about SUSY first to orient you.