Instructor

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Lectures

M, W 9:30—10:50 Mayer Hall Addition 2623

Homework

Will be assigned during lecture, and solutions will be posted on the web at http://physics.ucsd.edu/students/courses/spring2009/physics203b/

Close cousins to the homework problems will appear on the mid-terms and final.

Grade

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Lecture notes will be posted on web at http://physics.ucsd.edu/students/courses/spring2009/physics203b/

Texts

1. *Classical Theory of Fluids* by Landau and Lifshitz
2. *Classical Electrodynamics* by Jackson

Outline

1. Boundary value problems in electrostatics and magnetostatics
2. Electromagnetic waves and geometrical optics
3. Drude model for $\varepsilon(\omega)$
4. Magnetic diffusion and skin depth
5. Wave guides
6. Radiation
Gaussian system

Start with (cm, sec, gm, dynes, erg, etc.) from mechanics.

Basic equations for E&M

\[ F = qE + q\mathbf{v} \times \mathbf{B} \]
\[ \mathbf{F} + \mathbf{J} \text{ are total force and total current} \]

\[ \nabla \cdot \mathbf{E} = 4\pi \rho \]
\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = \frac{4\pi}{c} \mathbf{J} - \frac{1}{c^2} \frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \]

Charge

1 dyne force on two identical charged particles at rest

\[ F = eE \]
\[ \nabla \cdot E = 4\pi \rho \]
\[ \nabla \times E = 0 \]

1 dyne = \( \frac{(\text{statcoulomb})^2}{\text{cm}^2} \)

\[ e = 1 \text{ statcoulomb} \]

\[ F = eE \]

\[ e = \frac{e^2}{r^2} \]

\[ 1 \text{ statcoulomb} = \frac{1}{1 \text{ cm}^2 \text{ dyne}} \]
current

\[ I = \frac{dq}{dt} \]

statampsère = \text{statcoulomb} \cdot \frac{1}{\text{sec}}

electric field

\[ E = \frac{F}{q} \]

dyne \text{statcoulomb} = (\text{statcoulomb}) \cdot \text{cm} = \text{statvolt cm}

electric potential

\[ V \]

capacitance

\[ C = \frac{Q}{V} \]

\[ \text{statcoulomb} \text{statvolt} = (\text{statcoulomb})^2 \text{cm} \]

dyne cm
Force between two current carrying wires

\[
\begin{align*}
\mathbf{F} &= \frac{\mu_0 I_1 I_2}{2\pi r} \\
\mathbf{F} \times \mathbf{B} &= \frac{\mu_0 I_1 I_2}{2\pi r} \\
\mathbf{F} \cdot \mathbf{B} &= 0
\end{align*}
\]

\[
\text{force} = \frac{1}{\text{length}} \left( \frac{2I^2}{r} \right)
\]

This experiment determines the constant \( \frac{1}{c^2} \), and then Maxwell's equations predict speed of light, or measured speed of light and Maxwell predict force/length.
Magnetic induction \[ F = q v \times B \]

1 Gauss is B-field that produces force of \( \frac{V}{c} \) dyne on 1 statcoulomb moving 1 to 1 field.

[ Gauss = dyne/(statcoul.]

Inductance \[ L = \frac{\varepsilon_0 M_1}{(\frac{dI}{dt})} \]

\[ \frac{\text{statcoul}}{\text{statamp/sec}} = \frac{\text{see}^2}{\text{cm}} \]

Magnetized or Polarized material

\[ P, \quad \text{Polarization} \]

\[ \text{dipole moment} = \frac{\text{statcoulamps}}{\text{cm}^2} \]
Electric Displacement

\[ D = \varepsilon E + 4\pi \mathcal{P} \]

\[ \nabla \cdot D = 4\pi \rho_f \]

\[ E = D_{\text{vac}} \text{ in vacuum} \]

\[ \text{in medium use} \]

\[ \frac{\text{gauss volt}}{\text{cm}} = \frac{\text{statcoul}}{\text{cm}^2} \]

Magnetization

\[ \text{dipole moment} = \frac{\text{statcoul}}{\text{cm}^3} \]

Magnetic Field

\[ B = H + 4\pi M \]

\[ \nabla \times H = \frac{4\pi \mathcal{P}_f}{\varepsilon} + \frac{1}{c^2} \frac{\partial D}{\partial t} \]
\( H = B \) in \text{vac.} \\
\text{in medium vac } H = \text{continuous} \\
\begin{align*}
\frac{H}{\text{vac}} & \rightarrow \frac{B}{\text{vac}} \\
H & = B_{\text{vac}}
\end{align*}

\text{Oersted = Gauss}

\underline{\text{Rationalized MKS units}}

\text{(m, sec, N, T, Joule, etc.) from much}

Basic equations for \( \text{EM} \)

\( \mathbf{F} = \mathbf{P} \cdot \mathbf{E} + \mathbf{J} \times \mathbf{B} \) \quad \mathbf{P}, \mathbf{J} \text{ are total change and current}

\( \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \cdot \mathbf{B} = 0 \)

\( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \) \quad \text{(1)}

\( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \) \quad \text{(2)}
Note that

\[ u_0 = 4\pi \times 10^{-7} \frac{Nt}{\text{Ampere}^2} \]

\[ \text{still to be defined} \]

\[ C^2 = \frac{1}{\mu_0} \text{ is experimentally determined constant} \]

Coulomb's law

\[ E(r) = \frac{1}{4\pi \varepsilon_0} \iiint d^3r' \frac{P(r') (r-r')}{|r-r'|^3} \]

\[ B catast + Savart \]

\[ B(r) = \frac{u_0}{4\pi} \iiint d^3r' \frac{j(r') \times (r-r')}{|r-r'|^3} \]
Current defined through magnetic force

\[ \mathbf{\nabla} \times \mathbf{B} = \mathbf{u}_0 \mathbf{J} \]

\[ \mathbf{\nabla} \cdot \mathbf{B} = 0 \]

\( \mathbf{E} = \mathbf{\nabla} \times \mathbf{B} \)

\[ \mathbf{F} = \mathbf{p} \mathbf{E} \]

\[ \mathbf{F} = \frac{1}{2\pi \epsilon_0} \frac{(\mathbf{E}^2)}{r^2} \]

\( r = 1 \text{ m} \)

\( I = 1 \text{ ampere} \)

\[ \text{force} = \frac{2 \times 10^{-7} \text{ N}}{\text{m}} \]

1 ampere in wires separated by 1 m produces \( 2 \times 10^{-7} \text{ N/m} \) force

Charge

Condensate = Ampere see

To can be determined experimentally by measuring force between two charges (or by measuring speed of light)

\( c \approx 2.998 \times 10^8 \text{ m/s} \)
\[ F_\text{electric field} \quad F = \frac{q E}{m} \]

\[ \frac{N\text{m}}{C} = \frac{\text{Volt}}{m} \]

\[ \phi \quad \text{electric potential} \]

\[ \text{Volt} = \frac{\text{Joule}}{\text{Coul}} \]

\[ C \quad \text{capacitance} \]

\[ \text{Farad} = \frac{\text{Coul}}{\text{Volt}} \leftrightarrow \left( \text{compare in fact that Coul is not expressed in terms of units for force or length, so capacitance does not reduce to length} \right) \]

\[ B \quad \text{magnetic induction} \quad B = \text{vortex} \]

\[ \text{tesla} = \frac{N\text{m}}{\text{Coul} \cdot \text{m} \cdot \text{sec}} \]

\[ \text{magnetic flux} \quad \Phi = \oint B \cdot ds \]

\[ \text{weber} = \text{tesla} \cdot \text{m}^2 \]
Inductance

\[ \text{EMF} = L \frac{di}{dt} \]

Henry = \frac{Volt \sec}{amp}

Homework

1. Convert the following to Gaussian units:
   Volt, coulomb, henry, tesla, farad

Polarized and magnetized media

\[ \mathbf{b} = -\nabla \cdot \mathbf{P} \]
\[ \mathbf{J}_b = \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M} \]

\[ \nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = \mathbf{f}_0 \]
\[ \nabla \times \left( \frac{\mathbf{P}}{\varepsilon_0} - \mathbf{M} \right) = \mathbf{f}_0 + \frac{\partial}{\partial t} \left( \varepsilon_0 \mathbf{E} + \mathbf{P} \right) \]

\[ \mathbf{H} \text{ units amp/m} \]

\[ \mathbf{D} \text{ units coul/m}^2 \]
\[ \nabla \cdot B = 0 \quad \nabla \times E = -\frac{\partial B}{\partial t} \]

all constants gone

NBS units + atomic physics

batteries tend to have potentials of order Volt.

few meters of thin copper wire

\[ R = \text{ohm} \]

\[ I = \frac{\text{amp}}{\text{volt}} \]

\[ I = \frac{V}{R} \text{ amp} \]
Boundary value problems

J. Chapter 1, 2.3
L&T Vol.8, Chapter 1.2
Braun, Chapter 3

Uniqueness theorem

(motivate)

\[ \nabla \cdot D = 4\pi \rho \]

\[ D = \varepsilon(\sigma) \varepsilon \quad \varepsilon = -\nabla \phi \]

\[ \nabla \cdot \varepsilon(\sigma) \nabla \phi = -4\pi \rho \phi_d \]

Consider two solutions \( \phi_1(x) + \phi_2(x) \).
\[ \nabla \cdot (\nabla \phi_2 - \phi_1) = 0 \]

\[ \nabla \cdot \nabla \phi = \varepsilon (\nabla \phi)^2 + \mu \nabla \cdot (\nabla \phi) \]

\[ \int \int \phi \in \nabla \phi = \int \int \phi \in (\nabla \phi)^2 \]

Note that \( \varepsilon(\varepsilon) \geq 1 \) for thermal equil (L+L Vol 8 page 63)

1. Dirichlet b.c.

\[ 0 = \nabla \phi = \nabla \phi_2 - \nabla \phi_1 \text{ in } V \]

\[ 0 = \phi = \phi_2 - \phi_1 \text{ on } S \]

\[ \therefore \phi_1 = \phi_2 \text{ in } V \]
2. Neumann b.c.

\[ 0 = \nabla \phi = \nabla \phi_2 - \nabla \phi_1 \text{ in } V \]

if \( 0 = \hat{\mathbf{n}} \cdot \nabla \phi_2 - \hat{\mathbf{n}} \cdot \nabla \phi_1 \text{ on } S \)

modified Neumann b.c.
(parappropriate for conductors with total charge specified)

Any method that yields self-consistent.

Specify \( \Phi = \text{const.} \) on \( S_i \) and \( 4 \pi \Phi = - \oint \mathbf{E} \cdot \mathbf{dS} \)

\[ 0 = \nabla \Phi = \nabla \phi_2 - \nabla \phi_1 \text{ in } V \]

\[ \Phi = \text{const.} \text{ on } S_i \text{ and } \]

\[ 0 = \oint \mathbf{E} \cdot \mathbf{dS} = \oint \nabla \phi_2 \cdot \mathbf{dS} - \oint \nabla \phi_1 \cdot \mathbf{dS} \]

for all \( i \)

for all \( i \)
method of images (J. Chapter 2)

point charge near grounded plane conductor

\[
\phi(x, y, z) = \frac{q}{\sqrt{(x-d)^2 + y^2 + z^2}} \quad \text{and} \quad \frac{q}{\sqrt{(x+d)^2 + y^2 + z^2}}
\]

\[\phi = 0 \quad \text{for} \quad x=0 \quad \text{and} \quad \phi \to 0 \quad \text{as} \quad r \to \infty
\]

discuss surface charge and lines of force

Read sec 4.4 for point charge in front of dielectric interface
Work required to remove particle to \( \infty \)

Let particle be located at \( x \) (i.e., \( d=x \))

\[
F(x) = -\frac{q^2}{(2x)^2}
\]

\[
\text{Work} = \int_{x}^{\infty} -\frac{q^2}{2x^2} \, dx = \frac{q^2}{2x^2} \int_{x}^{\infty} \, dx
\]

\[
= \frac{q^2}{4x} = \frac{1}{2} \left( \frac{q^2}{2x} \right)
\]

Factor of \( \frac{1}{2} \) because no work done on mass change.

Alternately, field energy fills \( \frac{1}{2} \) of space
point charge near conducting sphere

\[ \nabla^2 \phi = -4\pi q \delta(r - d) \]

\[ q' = -\frac{q}{D} \]

\[ d' = \frac{D(a^2)}{D} \]

outside sphere

\[ \phi(r) = \frac{q}{|r - D|} - \frac{q}{|r - D^2/2|} \]

\[ \frac{q}{|r - D^2/2|} = \frac{q}{|D - r/2|} \]

\[ |D| = 0 \]

\[ \therefore \phi(a) = 0 \] also \[ \phi(r) \to 0 \] as \[ r \to \infty \]

discuss surface charge on sphere and lines of force

let \( \nabla \times \mathbf{E} \) dielectric \( \epsilon \)
Suppose that the condition $\Phi(a) = 0$ is replaced by the condition that net charge on sphere is $\Phi$,

$$\Phi(r) = \frac{a}{r - D} \left( \frac{\Phi}{r^2} \right) + \phi + \frac{\Phi}{r^2}$$

Discuss

$$\oint \nabla \phi \cdot d\mathbf{s} = 4\pi \left[ \Phi + \frac{\Phi}{D} - \frac{\Phi}{D'} \right]$$

electrode charges near gold atom

line charge near grounded conducting cylinder

let $\lambda' = -\lambda$

$$d = \frac{a^2 D}{D'}$$
outside cylinder

\[
\phi(\xi) = -2\pi \ln|\xi - D| + 2\pi \ln\left|\frac{\xi - \frac{a^2 D}{D^2}}{\frac{a}{D}}\right|
\]

\[
= -2\pi \ln\left|\frac{\xi - D}{\frac{a}{D}}\right| - \ln\left|\frac{\xi - \frac{a^2 D}{D^2}}{\frac{a}{D}}\right|
\]

\[
\Rightarrow \phi(\xi) = 0 \text{ for } |\xi| = 0
\]

\[
\lim_{|\xi| \to \infty} \phi(\xi) = 2\pi \ln\left(\frac{a}{D}\right) \quad \text{not localized charge dist.}
\]

The condition \( \phi = 0 \text{ at } \infty \) is replaced by \( \phi \) remains finite at \( \infty \). For example, this eliminates the

\[
\pi' \ln \frac{\xi}{a}
\]

Discuss surface charge and

lines of force.

let \( \nabla \cdot E = 0 \)
Homework

(2) J. 2.1
(3) J. 2.3
(4) J. 2.9
(5) J. 2.10 read sec. 2.5 (Sphere in uniform E-field)

Example steady state current

Salt water $J=0, E$

Large copper sphere

Copper $\gg$ Salt water

find current dist. neglect field of wire
\[ E = -\nabla \phi, \quad J = \sigma E \]

\[ 0 = \frac{\partial \phi}{\partial t} = -\nabla \cdot J = +\sigma \nabla^2 \phi \]

\[ \oint J \cdot ds = I \quad \text{small sphere} \]
\[ \oint J \cdot ds = -I \quad \text{large sphere} \]

\[ \phi(r) = \frac{I}{4\pi \sqrt{r-D}} - \frac{I \frac{q}{\beta}}{4\pi \omega (r - \frac{D\beta}{\omega})} \]

\[ + \frac{I \frac{q}{\beta} - I}{4\pi \omega |r|} \]

due to images in sphere

Images
 Charged wire above conductor

\[ E_t = 0 \]
\[ E_n \neq 0 \]

Current carrying wire above superconductor

\[ B_n = 0 \]
\[ B_t \neq 0 \]
Use vector potential for current-carrying wire

\[ \nabla^2 A_2 = -\frac{4\pi}{c} J_2 = -\frac{4\pi I L \delta(x) \delta(y-h)}{c} \]

At surface require

\[ 0 = B_y = \hat{y} \cdot \nabla \times \hat{z} \mathbf{A}_2(x, y) = \frac{\partial A_2}{\partial x} \]

\[ A_2 = \text{const. on surface, so} \]

\[ A_2(x, y) = \phi(x, y) \]